

# The Century of Strong Gravity

January 26, 2015

## Summary

Einstein's General Relativity is one of Humanity's greatest intellectual achievements and, even one century after its formulation, continues to challenge scientists. In this theory, gravity is explained as an effect of the curvature of spacetime and not in terms of long-distance forces. General Relativity predicts the existence of "entities" made solely from pure spacetime fabric: black holes and gravitational waves. Gravitational waves carry information about the most violent astrophysical processes and are all around the Universe. However, their message has so far been lost in space because of the difficulty in detecting them. In the next few years, these missing predictions of General Relativity will be finally observed, perhaps shedding light on the unknown Universe and on the quantum side of gravity.

# 1 One Hundred Years of Gravity

The latest Christopher Nolan’s movie, *Interstellar*, is about a future human civilization able to undertake cosmic travels to black holes using special shortcuts, “wormholes”. Science-fiction as it might seem, *Interstellar* screenplayers – who happen to be the Nolan brothers – have worked side by side with Kip Thorne, a professor of Theoretical Physics at the California Institute of Technology and one of the fathers of modern General Relativity, the theory that explains what wormholes and black holes are and how they form in the Universe.

Thorne’s contribution is to ensure that the movie – starring Matthew McConaughey and Anne Hathaway among others – doesn’t contain scenes that would make Albert Einstein cringe. Does this mean that travel agencies are about to sell (roundtrip!) tickets to a black hole? Not quite, but in a few years from now, theoretical physicists and astronomers will be able to study them as never before. The scientific payoff of these studies will largely overcome *Interstellar*’s box-office, with all due respect to Mr. Nolan!

Einstein’s theory of General Relativity will celebrate its 100th anniversary in 2015, but it is far from being fully understood. In fact, the most astonishing predictions of the theory – such as the existence of black holes and gravitational waves – have not been confirmed yet, but novel detectors and space missions hold the promise to do so in the next few years.

What are gravitational waves? And what do they have to do with black holes? Before answering these questions, we need to explain what General Relativity is. For various reasons, General Relativity is a revolutionary idea. For all other fundamental interactions (like electromagnetism governing the dynamics of electrons and photons or the “strong interaction” governing quarks inside protons and neutrons) *spacetime* is the arena, a helpless spectator that only dictates “where” and “when” the interaction is taking place. Einstein understood<sup>1</sup> that the fundamental principles of gravity forces spacetime to enter the game: spacetime reacts to any form of energy and is deformed, pretty much like a sheet of elastic rubber is deformed by a heavy sphere rolling on top of it, as shown in Figure 1. In other words, spacetime becomes a dynamical entity which makes the very

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<sup>1</sup>Another of Einstein’s revolution was the use of *gedankenexperiments* – German for “thought experiments”. In a theoretical tour-de-force, Einstein, like no other physicist before or after him, deduced the equations governing gravity only by logical conclusions.

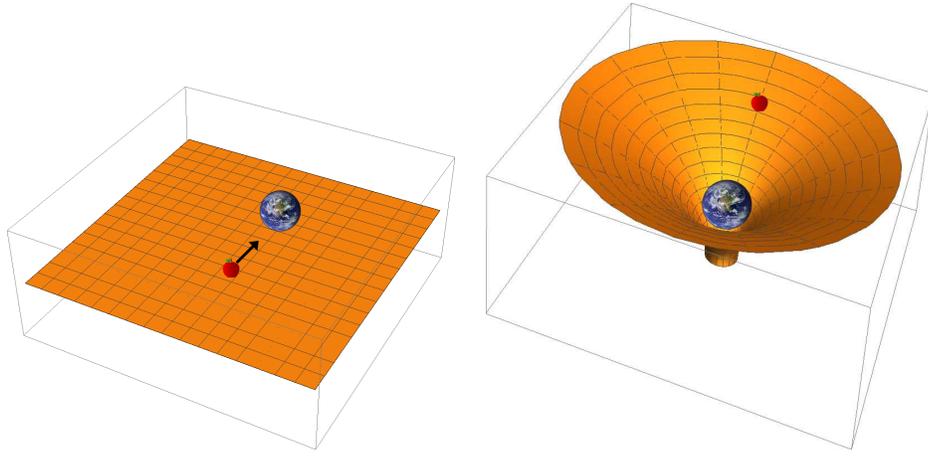


Figure 1: Gravity from Newton’s and Einstein’s perspectives. On the left, the apple falls because of the attractive force Earth exerts on it. This force is governed by the famous Newton’s law: is proportional to the mass of the apple, to the mass of Earth and is inversely proportional to the square of the distance between them. On the right, the apple’s straight trajectory is deformed by the curvature of spacetime. The deformation depends on Earth’s mass. Not only space is deformed, but also time: a clock attached to the apple runs differently as the apple approaches Earth’s surface.

concept of forces— upon which Newton’s theory is based—superfluous. Einstein predicted that Newton’s apple didn’t fall because of the force Earth exerts on it, but rather... it didn’t fall at all! Earth’s big *mass curves spacetime* in such a way that the apple can freely “roll down” *as if* it was accelerated.

Einstein’s greatest revolution was to understand gravitational interactions in geometrical terms, introducing the concept of *spacetime curvature*. In fact, his theory is often referred to as *geometrodynamics* because it governs how the geometry of spacetime responds to the presence of moving bodies.

Despite such drastically different views, Einstein’s predictions agree with Newton’s theory in everyday life. Near the Earth’s surface, spacetime is almost flat and the differences between General Relativity and Newton’s law of universal gravitation are very small (but without Einstein’s theory, a GPS would not work!). However, celestial objects much more compact than the Earth – like our Sun, white dwarfs and neutron stars – are so massive that spacetime

curvature becomes important. Furthermore, General Relativity incorporates Einstein’s other great achievement, the famous  $E = mc^2$  relation, which tells us that mass and energy are two sides of the same coin: it is not only mass that curves spacetime, any form of energy does so. For example, two compact stars orbiting each other move almost at the speed of light and their kinetic energy is one of the largest sources of spacetime curvature.

As the famous physicist John Wheeler has put it: “*Matter tells space how to curve, and space tells matter how to move*”. This sentence underlies both the beauty and the beast of General Relativity. Such elegant interplay makes it extremely challenging to solve Einstein’s equations, as the motion of bodies and curvature of spacetime are indissolubly connected. From such subtle exchange, two entities emerge which are solely made of spacetime fabric: black holes and gravitational waves.

## 1.1 Black holes

Nobody has seen a black hole and yet almost everybody has a rough idea of what a black hole is. In fact, most people would argue that nobody will ever see a black hole, just because... it’s a hole!<sup>2</sup>.

A black hole is a region of spacetime so highly deformed that no signal – no matter how fast – can escape from its interior. Its border is called *event horizon*. The reason is simple: since not even light can escape from the event horizon, *what happens inside a black hole, stays in the black hole*. In other words, the interior of a black hole is beyond our “horizon”, we can never be affected by any event inside the event horizon of a black hole.

The black hole concept was introduced only in the 1960s, but already in the late 18th century “dark stars” were explored using Newton’s theory. Indeed, every high-school student can repeat Michell’s 1783 original argument and compute the escape velocity of an object at the surface  $R$  of a star with mass  $M$ . This velocity is

$$v_{\text{esc}}^2 = \frac{2GM}{R}, \quad (1)$$

where  $G$  is Newton’s constant. Because light moves at a speed  $c \approx 300000\text{km/s}$ , the condition that not even light escapes from the

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<sup>2</sup>Fortunately, this is not exactly true. As we will explain later, there are various ways to “see” a black hole, other than watching Interstellar, of course...

surface of a “dark star” imposes a limit on the star’s radius, which must be smaller than

$$R_{\text{Schwarzschild}} = \frac{2GM}{c^2}. \quad (2)$$

Curiously, this is precisely the radius of a black hole in Einstein’s theory, known as Schwarzschild radius!<sup>3</sup>

Thus, spacetime can be deformed so much that not even light can escape, but what can cause such enormous deformation? The Schwarzschild radius gives us the answer: our Earth (whose mass is roughly  $M_{\oplus} \approx 6 \times 10^{24}\text{kg}$ ) should be compressed in one centimeter to become a black hole!

It seems therefore impossible that such compact objects can ever be formed. However, gravity has a unique feature that makes it possible – and in fact likely – to squeeze matter in such small regions. Namely, gravitational interactions are always *attractive*. When matter starts accumulating towards the center of a big star, it becomes more compressed as more matter is attracted. Sometimes, (as in the case of Earth) the pressure at the surface is sufficient to balance the effect of gravity and the object remains in equilibrium. However, in some other situations gravity can be so strong that no material – no matter how solid and incompressible – can sustain its own gravitational weight. The situation is similar to a house of cards: if a critical mass is reached, the house cannot sustain its own weight and collapses. In stars, this process is called gravitational collapse and in many cases the leftover (after a lot of energy is released in what is called a “supernova explosion”) is precisely a black hole.

A supernova explosion is one of the brightest events in the Universe; during such explosions a single star can outshine the entire galaxy. This is one of the reasons we have observed many supernovae even when they explode millions of light years away from us<sup>4</sup> and also one of the reasons we believe that millions of black holes exist in our galaxy.

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<sup>3</sup>Karl Schwarzschild’s discovery has a fascinating history. Schwarzschild found its famous solution while serving in the German army during World War I, only a few months after Einstein’s had published his theory. It’s like quantum gravity will be discovered by a soldier serving in Syria nowadays, how impossible can it be? Unfortunately, Schwarzschild died the year following his discovery from a disease he developed while at the Russian front.

<sup>4</sup>One famous supernova is SN 1006 and was seen on Earth in the year 1006. From its apparent magnitude, it’s considered the brightest stellar event in recorded history. Curiously, some theories have argued that the star of Bethlehem – that had supposedly revealed the birth of Jesus to the Biblical Magi – was in fact a supernova.

One huge black hole is lurking right at the center of our Milky Way. By studying the orbits of stars around the center of our galaxy, astronomers have concluded that they move around a huge dark object of about 4 million solar masses. No star can ever be so massive without collapsing and, despite decades of efforts to provide alternative explanations, a black hole remains the most *conservative* hypothesis for whatever it is that lies at the Milky Way's center (and, as it turns out, of almost *any* other galaxy).

Although black holes do not shine, they can talk, and very loudly so. While other objects (like stars, planets, and even people) can be seen by the electromagnetic radiation they emit, black holes can be detected by the gravitational radiation they produce.

## 1.2 Gravitational waves

Spacetime is like an elastic sheet of rubber in which objects move freely. If the objects in Figure 1 remain at rest, nothing special happens. However, imagine that an object moves on the deformed rubber. This motion generates *waves* that propagate on the surface of the rubber at a characteristic velocity that depends on the properties of the material. Something similar happens when a boat sails on a calm lake: surface waves propagate during the motion. In a similar fashion, gravitational waves are oscillations of the spacetime curvature that transport energy. In this sense they are similar to acoustic waves (sound) or electromagnetic waves (light) but they cannot be directly heard nor seen. However, they can be detected: during their passage, they modify the *distance* between two objects and even the time measured by clocks!

Gravitational waves are a genuine prediction of General Relativity and do not exist in Newton's theory, where gravity is an instantaneous interaction. Imagine that our Sun suddenly disappears. In Newton's theory, Earth and the other planets would instantaneously feel this absence and would immediately stop orbiting around the center of our solar system. Einstein's theory, however, incorporates all the principles of relativity, including that no information propagates faster than light in vacuum. Light would take about 8 minutes to travel from the Sun to Earth and to tell us that the Sun disappeared. The "messengers" bringing this information are precisely the gravitational waves which in fact propagate at the speed

of light, a good  $c \approx 300000\text{km/s}$ . The (hypothetical, because have not been detected yet) particles associated to gravitational waves are called *gravitons* in analogy with the photons that are instead the corpuscular description of electromagnetic waves.

As we discuss below, gravitational waves are generated copiously by massive objects, like neutron stars, black holes and compact binary systems. These objects emit most of their enormous energy in gravitational waves. For example, a binary system of two black holes loses energy through gravitational-wave emission, and its orbit shrinks until the two objects merge. When they are about to merge, the luminosity in gravitational waves is about

$$L \approx 10^{52}W. \quad (3)$$

This is a ridiculously large number, outshining the entire Universe! For comparison, the luminosity of the Sun is “only”  $L_{\odot} \approx 3.8 \times 10^{26}\text{W}$ .

It might therefore come as a surprise that, almost one century after they have been predicted, gravitational waves have not been directly observed yet. The reason is simple: first, such catastrophic events like black-hole mergers, supernovae explosions, and other “loud” processes typically occur very far from us, so their enormous energy is spread over a large portion of the universe; secondly, once produced gravitational waves interact only very feebly with matter, and their detection on Earth is a very challenging task.

On the other hand, precisely because gravitational waves interact so weakly, they can travel the entire Universe without being perturbed or quenched. This is very important for astrophysics, because the signal they carry is totally uncontaminated, for example it does not suffer from obstruction like typical electromagnetic signals. We will come to this point later, when discussing detection of gravitational waves, an epic task finally within the reach of the new generation of detectors. Before doing that, let’s have a look at how these spacetime ripples propagate and at the sources producing them.

## 2 Properties of gravitational waves

Imagine that you are in your gym class holding a hula hoop and suddenly a gravitational wave passes perpendicularly through it.

Would you be able to “see” it? Well, actually not, because the effect of the wave on the hula hoop would be extremely small<sup>5</sup>. But let’s forget for a moment that this effect is small. Using a magnifying glass, what you would see is depicted in Figure 2, where time flows in the horizontal axis. There are two different ways you can see the wave deforming the hoop. You’ll see it oscillating Either in a + shape or in a × shape. These are called the two different *polarizations* of gravitational waves<sup>6</sup>

Consider a hoop with diameter given by  $L$  and that the wave has an amplitude given by  $h$ . Due to the interaction of the hoop with the wave, its diameter will suffer a small variation  $\delta L$  such that

$$\frac{\delta L}{L} = h. \quad (4)$$

We can easily see from this equation that the larger the wave’s amplitude  $h$  the larger the deformation  $\delta L$  that the hoop will suffer. For a typical hula hoop size of  $L = 1$  meter and a gravitational wave with amplitude  $h = 0.001$ , the hoop will suffer deformations of the order  $\delta L = 1$  millimeter. What  $h$  is actually measuring is the way the trajectories of different points in the hoop are affected by the curvature of spacetime. In fact,  $h = 0.001$  is already a huge deformation, and if gravitational waves with these amplitudes were common, we would be able to detect their effects with the naked eye. Unfortunately, as we discuss below,  $h$  is extremely small even for extremely powerful sources, which is one of the reasons why the detection of gravitational waves is so difficult.

## 2.1 Sources

Gravitational waves are sourced by almost any kind of motion. The intensity of these waves will normally depend on the source (just as the intensity of electromagnetic waves depends on the antenna that generates them).

For example, the typical amplitude of the gravitational waves emitted when two objects with equal mass  $M$  orbit each other in a circular orbit at a distance  $r$  is

$$h \sim \frac{G^2 M^2}{c^4} \frac{1}{r D}, \quad (5)$$

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<sup>5</sup>If you do see it, you are probably experiencing a strong earthquake, so stop reading this and go hide in a secure place.

<sup>6</sup>In general the wave is not polarized and you would see a combination of both.

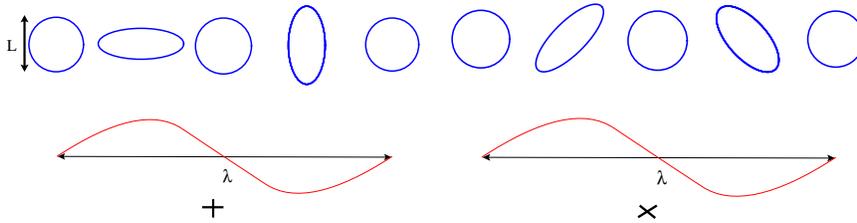


Figure 2: The deformation of a hula hoop due to the passage of a gravitational wave perpendicular to this sheet. There are two kinds of “polarizations”: the + on the left and the  $\times$  on the right. These two polarizations are a consequence of gravitons being massless, like photons. If gravitons were massive, we would see five different polarizations instead of two.

where  $D$  is the distance of the observer.

The formula above tells us something very familiar from our everyday lives, the farther we are from the source, the smaller the intensity of the wave we can detect<sup>7</sup>.

If we want to detect gravitational waves, why don’t we just build a powerful source here on Earth? Imagine a source consisting of two huge weights with one tonne each, and set them in a circular motion, 1 meter from each other at a rate of 1000 times per second. If we assemble our detectors at 300 kilometers, we would hypothetically detect waves with amplitude  $h \sim 10^{-49}$ . This is way too small for any conceivable detector to be able to see them! In fact there is not much that we can do here on Earth. But as the first astronomers did, we have to look at the skies and hope to find something there which might work as a powerful source.

### 2.1.1 Binary systems of black holes and neutron stars

One of the most promising systems of detectable gravitational radiation are binaries of black holes or neutron stars<sup>8</sup>. These systems are very promising because they are highly compact and can move at very high speeds. When two black holes collide their distance  $r$  is of the order of their Schwarzschild radius. In this case, assum-

<sup>7</sup>Turn on your TV and try to measure the sound intensity as a function of the distance as you walk away from it. In a perfect scenario, in which the sound does not dissipate for other reasons, you would find the same  $\sim 1/D$  relation.

<sup>8</sup>Neutron stars are the most compact stars that exist without collapsing into a black hole. They are held together by quantum mechanical effects and have huge densities. For example a teaspoon of neutron star material can be as heavy as 900 times the mass of the Great Pyramids!

ing the black hole mass to be about 1000 times that of our Sun<sup>9</sup>, Equation (5) gives us the amplitude of the gravitational waves at a distance  $D$ ,

$$h \approx \frac{10^4 \text{km}}{D}. \quad (6)$$

Therefore, an observer starring at the merger about 400000 kilometers away (this is roughly the Earth-Moon distance) from the black holes would see  $h \approx 0.001$  and its hula-hoop would shrink by one millimeter! This would happen to any object on Earth if our Moon were just replaced by a massive black hole binary system (imagine how strange the world would look like in this case!). Clearly, this is an unrealistic example. If we were located at 1Megaparsec<sup>10</sup> from the system, the amplitude of gravitational waves would be only  $h \approx 10^{-17}$ . This is a very small number: it's possible that such wave is traversing you at this very moment, but you are being deformed by only a fraction of a proton radius.

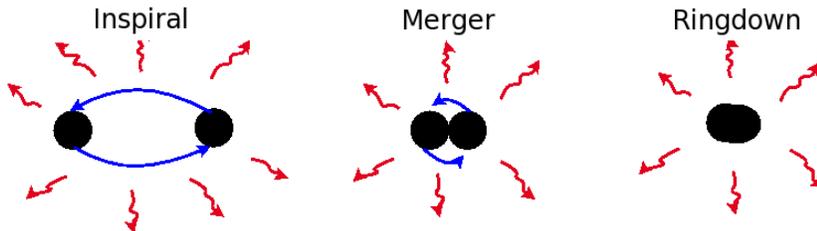


Figure 3: Three phases in the life of a binary system of compact objects. In the “spiral phase” the objects are well separated and orbit around each other, emitting gravitational waves and slowly inspiralling. In the second part of their life, the “coalescence phase”, they have lost enough energy and collide forming one single object – typically a black hole – which slowly emits waves until it reach an equilibrium state (“the damped phase”), similarly to the ringing of a bell.

There is however a way around this, which is looking for the *indirect* effects of the emission of gravitational waves. These waves carry energy away from the system. Since the system is losing energy, the radial separation between the two black holes will decrease with time. Thus, even if we cannot detect gravitational waves directly, we can look at the orbits of compact binaries and see if they shrink according to General Relativity.

<sup>9</sup>Astronomers have observed black hole candidates as massive as one billion solar masses.

<sup>10</sup>A megaparsec is equivalent to 3 hundred thousand light-years or  $3 \times 10^{22}$  meters.

Hulse and Taylor did precisely this, by monitoring the pulsar<sup>11</sup> PSR 1913+16 over two decades (starting in 1974) while this object was orbiting another neutron star. They found that the orbital period decreased as predicted by General Relativity to a remarkable precision. These observations pioneered a new era in pulsar astronomy and – as a recognition of the impact that they had in science – were awarded a Nobel prize in Physics in 1993. In the decades that followed this discovery it has become an acquired fact that gravitational waves exist and could one day become important astronomical tools to observe our Universe.

The importance that gravitational waves might play in astronomy is mostly due to the unique information that they carry about the source. Theoretically, we now know exactly how the radiation emitted by this binary would look like. As we said, the system will lose energy in the form of radiation, which causes the two objects to slowly inspiral towards each other until they eventually collide and become one single final object. For example, the neutron stars of Hulse-Taylor binary system will collide in approximately 240 millions of years from now. This process has three different phases which we illustrate in Figure 3. The radiation emitted by the system will have a very particular signature of these different phases. The form of this wave is shown in Figure 4 and it is obtained by solving Einstein’s equations.

These “mergers” are the most powerful processes in our Universe. For a system of two black holes with a thousand solar mass each, the last ten orbits before the collision lasts only for 10 ms. If we observed this system at a distance of 150km we would be stretched or compressed by 3mm when the wave passes (unfortunately this kind of systems are light-years of distance from us). Furthermore, the energy carried by the wave corresponds to the energy of 100000 Earth masses. This is a gigantic luminosity (energy received per second), much larger than the luminosity of the whole Universe that we currently observe!

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<sup>11</sup>A pulsar is a rotating neutron star emitting a beam of electromagnetic radiation. Since we can only observe this radiation when the beam is in our line of sight, what we observe is a very periodic emission of radiation, hence the name pulsar (“pulsating star”).



First, primordial gravitational waves are produced by quantum fluctuations in the Early Universe and are generically very weak to be detected at the present epoch. However, they can be enormously amplified within a theory called *cosmological inflation*. Inflation predicts the Universe has undergone a phase transition right after the Big Bang, in which spacetime started expanding exponentially for an extremely short time ( $10^{-32}$  seconds) and amplifying any small fluctuation to detectable size<sup>13</sup>. For this reason, a detection of primordial gravitational waves from the Big Bang would automatically be the first evidence for inflation, something that cosmologists and particle physicists are after since the late 1970s.

Secondly, such detection would itself be a second independent confirmation of the existence of gravitational waves.

Finally, classical General Relativity and inflation alone cannot explain why an experiment like BICEP2 could have detected such signal: the very production mechanism to produce such spacetime ripples is of a quantum nature. The energy scale of such effects (some  $10^{25}$  electron-volts) is much larger than the energy currently produced in particle accelerators. Therefore, the very fact that we can detect such effect is already a confirmation of the quantum nature of gravity, the Holy Grail of all theoretical physics.

The measurement of BICEP2 would be a breakthrough in theoretical physics, astrophysics and cosmology and, as such, requires the most precise and scrupulous verification. The interpretation of the polarization of the cosmic microwave background in terms of primordial gravitational waves is currently debated. At the time of writing, two of the major experiments – PLANCK and BICEP – are jointly analyzing their data to come to a final answer, which will be probably released in 2015.

### 3 Gravitational-wave detectors

Because the most powerful sources of gravitational waves are very far from us, the amplitude of the gravitational waves when they reach the Earth is tiny. Theoretical computations and astrophysical

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<sup>13</sup>The success of inflationary models consists in explaining other characteristics of the Universe that were already observed in the past, such as its flatness, homogeneity and isotropy. Essentially, inflation provides a dynamical mechanism from which – starting from generic initial conditions – the universe emerges as homogeneous and isotropic right to the level that we now observe.

observations predict that the amplitude of a typical gravitational wave on Earth will be roughly  $h \approx 10^{-21}$ : a rod as long as the Sun-Earth distance would shrink by only one atomic size during the passage of such weak gravitational wave!

Impossible as it might seem, there are detectors that can achieve such unbelievable sensitivity. These instruments are called *gravitational wave interferometers*, because they use the principles of interferometry to measure tiny differences between two lengths. In very simple terms, these detectors are made of two perpendicular “arms” of equal length (typically a few kilometers) with one laser beam each. Thanks to a series of “mirrors”, the two beams are made to interfere and from the interference pattern one can measure very precisely tiny deviations in the relative size of the arms. Because the arms are perpendicular to each other, a gravitational wave with polarizations as in Figure 2 would deform each arm differently and can potentially be detected.

Believe it or not, gravitational-wave detectors *do work*. At the moment, there are two large interferometers in the U.S. (LIGO experiment), and a similar detector (Virgo) operated by the European Science Observatory in Italy. Virgo and LIGO are now being upgraded to increase their sensitivity by a factor ten and they will return to observe the gravitational Universe in 2016-2018. By then, the enhanced sensitivity would allow them to detect binary inspirals on a monthly basis. Meanwhile, a third detector (KAGRA) is under construction in Japan and design studies are ongoing for a third-generation detector, so-called Einstein Telescope.

Things are going on not only on Earth, but also in space. In recent years there has been a flurry of activity related to space-based missions. An updated version of the original proposal for a Laser Interferometer Space Antenna (LISA) is currently supported by the European Space Agency and is expected to flight in 2030s. Space-based detectors will be sensitive to low-frequency gravitational waves as those emitted by supermassive black holes (like the one at the center of our galaxy).

Finally, complementary to laser interferometers, the prospect of detecting gravitational waves using the signal from various pulsars as a huge interferometer on a cosmic scale (so-called pulsar timing arrays) has been recently explored, and it is not impossible that a gravitational-wave signal is already lurking in data astronomers are

currently analyzing.

## 4 Gravitational-wave astronomy: a new window on the Universe

The ensemble of observational facilities promises to give birth to precision gravitational-wave astronomy and will open a totally new window onto the Universe, complementing the wealth of information from present electromagnetic observations and recent cosmological surveys. This global effort is motivated by the huge scientific potential of gravitational-wave astronomy. Because gravitational waves interact very feebly with matter, they carry direct information from the interior of neutron stars, where the density of matter is higher than that of a nucleus and physical processes are not well understood. Furthermore, from this new window we will be able to *hear the sound of spacetime* from loud events like the merger of two black holes, which can give invaluable information on Einstein’s gravity. Thousands of systems that involve supermassive black holes are copious gravitational-wave emitters in the low-frequency band that will be accessible with eLISA. The exquisite precision of such mission would allow for a precise catalog of distant galaxies, tests of General Relativity and would even shed light on the very origin of the Universe.

Last but not least, by detecting gravitational waves we could finally test Einstein’s theory in its most extreme configurations, where gravity is the dominant interaction. For instance, if we discover that gravitational waves don’t deform matter in the way depicted in Figure 2, it would mean that something has to change in General Relativity and that Einstein was not (completely) right after all.

Metaphorically, much alike the prisoners in Plato’s Cave, at the moment we are just observing the “electromagnetic shadows” of the gravitational Universe, but its “real shape” – emitted in gravitational waves – has so far been lost in space. As in the worst nightmare, it seems like the Universe is constantly and loudly talking to us, we can see its mouth opening but cannot hear any sound yet. Whatever this message contains, just receiving it will be yet another giant leap for mankind. Keep listening...